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Robust stabilization of a wheeled vehicle: Hybrid feedback control design and experimental validation †

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Abstract

In this paper, the problem of robust stabilization of a wheeled vehicle is addressed. The configuration (position and orientation) set of the vehicle is divided into two parts: global and local configuration sets. The novelty of this paper is the design of a hybrid feedback controller that assigns different objectives in the vehicle's global and local behaviors. Two Lyapunov functions for individual objectives are introduced that allow a hybrid feedback control law to pursue different objectives. In the global sense, it is important to reach the target point as quickly as possible, but once the vehicle reaches is near the goal, a precise maneuvering by rejecting disturbances including tire slippage and measurement noise becomes important. The asymptotical stability and robustness of the closed loop system are assured. The derived control law is validated by simulations and experiments using an autonomous forklift.

Keywords: Hybrid feedback control; Lyapunov function; Robustness; Stabilization; Wheeled vehicle

1. Introduction

The control problems of wheeled vehicles have been intensively studied in recent years and many control problems have been conducted [1-9]. One of the control problems of the autonomous wheeled vehicle is the ability to perform point to point motion (stabilization) where a desired goal configuration must be reached starting from a given initial configuration [10-15]. The problem becomes more challenging as a wheeled vehicle is a nonholonomic system (a system with nonholonomic constraints). Due to Brocket Theorem [16], a nonholonomic system cannot be asymptotically stabilized by using continuously differentiable and time-invariant control.

Several methods to stabilize a nonholonomic system via feedback control have been proposed in the literature. As reported in [17, 18], a stabilization method that provides a fast and natural performance path is conducted by transforming the configuration variables into navigation variables [19]. The navigation variables are the distance from the vehicle frame (a body coordinate system attached to the vehicle) to the target frame (a desired goal configuration of the vehicle), the angle between the vehicle-to-target (v-to-t) vector and the target frame, and the angle between the v-to-t vector and the current vehicle orientation.

Although the problem of stabilizing the nonholonomic system has been theoretically solved [20-24], most papers have assumed ideal condition (no input disturbances and measurement noises) on the wheeled vehicle system. In the implementation, the robustness issue against actuator disturbances and measurement noises deserves further attention. As indicated in [25], known feedback laws that globally exponentially stabilize the mobile robot with no input disturbances and measurement noises are not robust against small disturbances. Several methods have been proposed to study the robust stabilization of the nonholonomic system. Most of them focus on identifying parametric uncertainties and model errors [26, 27]. Other papers have attempted to overcome the fundamental robustness by implementing hybrid feedback control [28] and velocity scheduling control [29-32].

In this paper, a method to solve the robust stabilization of a wheeled vehicle with one driving-and-steering wheel in the rear based on a hybrid feedback control is presented. The kinematics of the vehicle is first reviewed and the configuration variables of the vehicle are reformulated in the form of navigation variables. The sets of the vehicle configuration are divided into two parts: the configurations that are close to the goal configuration (local configuration set) and are distant from the goal configuration (global configuration set). Two

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Lyapunov functions corresponding to each set are then defined. The Lyapunov function for the global configuration set is defined as the weighted norm of navigation variables. The Lyapunov function for the local configuration set is chosen such that the terms, which include disturbances, are eliminated. From each set, the control law is derived yielding a hybrid feedback control law. The control law yields asymptotical stability and robustness against small input disturbances and measurement noises. We show that the closed loop system using only the feedback control law derived from the Lyapunov function for the global configuration set is not robust against small input disturbances and measurement noises when the vehicle configuration is in the local configuration set. The proposed control scheme is studied by simulations of and experiments in the stabilization of a real autonomous forklift.

The contributions of this paper are as follows. First, a hybrid feedback control scheme to stabilize a wheeled vehicle is presented. Second, asymptotic stability and robustness against input disturbances and measurement noises of the closed loop system are achieved. Finally, the experimental testing of the stabilization of the wheeled vehicle is presented, which has appeared in only a few studies.

The paper is organized as follows. Section 2 presents the problem formulation. The kinematics of a vehicle with one driving-and-steering wheel in the rear with input and measurement disturbances is discussed and the robust stabilization problem is defined. Section 3 discusses the development of the hybrid feedback controller, the investigation of the asymptotic stability, and the robustness analysis of the closed loop system. Section 4 presents the simulations and experimental results of the stabilization of the autonomous wheeled vehicle. The conclusions are given in Section 5.

2. Problem description

Fig. 1 shows the vehicle model with two fixed wheels in front and one drivable-and-steerable wheel in the rear. It is assumed that the rotational motions of all the wheels of the vehicle are pure rolling with no slipping. The kinematics of such type of the vehicle is given as follows.

$$\dot{X} = v_{dr}\cos\theta\cos\delta\ , \quad (1)$$

$$\dot{Y} = v_{dr}\sin\theta\cos\delta\ , \quad (2)$$

$$\dot{\theta} = -v_{dr}/l\sin\delta\ , \quad (3)$$

where X, Y are the coordinates of the reference point O' (which the vehicle motions are generated in the global coordinate frame OXY and becomes the origin of the local coordinate frame $O'X'Y'$); θ is the orientation of the local coordinate frame $O'X'Y'$ with respect to the global coordinate frame OXY (in the counter clock-wise direction); and l represents the distance between the center of the rear wheel and the axis of the front wheels. The two control inputs are the driving velocity v_{dr} and the steering angle δ, both of which are applied at the rear wheel.

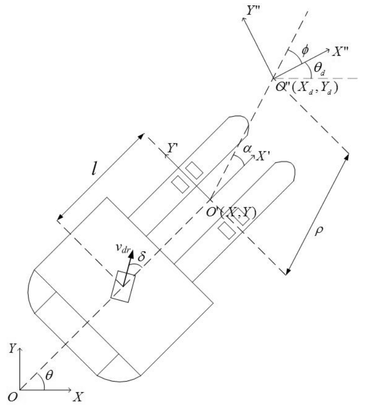


Fig. 1. The vehicle model.

A smooth stabilizing control law can be derived by transforming the configuration variables (X, Y, θ) into navigation variables (ρ, ϕ, α) which are defined as follows: ρ is the distance between O' and O'' (a given target point in the global coordinate frame attached to the target coordinate frame $O''X''Y''$), which is $\rho = ((X_d - X)^2 + (Y_d - Y)^2)^{1/2}$ where X_d, Y_d are the position of O'' in the global coordinate; ϕ is the angle made by the v-to-t vector (a vector connecting O' and O'') and the X''-axis of the target coordinate frame $O''X''Y''$, which is $\phi = \operatorname{atan2}(Y_d - Y, X_d - X) - \theta_d$; and α is the angle between the v-to-t vector and the X'-axis of the local coordinate frame $O'X'Y'$, which is $\alpha = \phi - \theta_e$, $\theta_e = \theta - \theta_d$, where θ_d is the orientation of the target coordinate to the X-axis from the global coordinate frame. In the rest of the paper, we assume that the goal desired configuration of the system is $(X_d, Y_d, \theta_d) = (0, 0, 0)$ which can also be expressed by $(\rho, \theta_e) = (0, 0)$. The kinematic equations in the navigation variables domain are written as follows.

$$\dot{\rho} = -v_{dr}\cos\alpha\cos\delta\ , \quad (4)$$

$$\dot{\phi} = v_{dr}/\rho\sin\alpha\cos\delta\ , \quad (5)$$

$$\dot{\alpha} = v_{dr}(\sin\alpha/\rho + \tan\delta/l)\cos\delta\ , \quad (6)$$

where $\dot{\theta}_e = \dot{\theta}$ has been assumed as θ_d is constant.

Let ε_X, ε_Y be the measurement noises of position (X, Y) and let ε_θ be the measurement noise of the orientation θ of the vehicle. The measurement values of the position $(\hat{X}, \hat{Y})$ and the orientation $\hat{\theta}$ are defined as follows: $\hat{X} = X + \varepsilon_X$, $\hat{Y} = Y + \varepsilon_Y$, and $\hat{\theta} = \theta + \varepsilon_\theta$, where $|\varepsilon_X| \le \varepsilon_X^{\max}$, $|\varepsilon_Y| \le \varepsilon_Y^{\max}$, and $|\varepsilon_\theta| \le \varepsilon_\theta^{\max}$; $\varepsilon_X^{\max}$, $\varepsilon_Y^{\max}$, and $\varepsilon_\theta^{\max}$ are the absolute maximum values of the measurement noises of the position (X, Y) and of the orientation θ, respectively. Let ε_ρ, ε_ϕ, and ε_α denote the state feedback disturbances of the navigation variables ρ, ϕ, and α, respectively: $\varepsilon_\rho = ((X_d - \hat{X})^2 + (Y_d - \hat{Y})^2)^{1/2} - ((X_d - X)^2 + (Y_d - Y)^2)^{1/2}$, $\varepsilon_\phi = \operatorname{atan2}(Y_d - \hat{Y}, X_d - \hat{X}) - \operatorname{atan2}(Y_d - Y, X_d - X)$, and $\varepsilon_\alpha = \varepsilon_\phi - \varepsilon_\theta$. The input disturbances of the driving velocity and steering angle are defined by ε_{vdr} and ε_δ, respectively; $|\varepsilon_{vdr}| \le \varepsilon_{vdr}^{\max}$ and $|\varepsilon_\delta| \le \varepsilon_\delta^{\max}$ where $\varepsilon_{vdr}^{\max}$ and $\varepsilon_\delta^{\max}$ are the absolute maximum of the input disturbances of the driving velocity and steering angle, respectively. With the existing of input disturbances, (4)-(6) are rewritten as follows.

$$\dot{\rho} = -(v_{dr} + \varepsilon_{vdr})\cos\alpha\cos(\delta + \varepsilon_{\delta}) , \quad (7)$$

$$\dot{\phi} = (v_{dr} + \varepsilon_{vdr}) / \rho \sin\alpha\cos(\delta + \varepsilon_{\delta}) , \quad (8)$$

$$\dot{\alpha} = (v_{dr} + \varepsilon_{vdr})(\sin\alpha / \rho + \tan(\delta + \varepsilon_{\delta}) / l)\cos(\delta + \varepsilon_{\delta}) . \quad (9)$$

It is assumed that the steering angle δ is in the range of $\delta + \varepsilon_{\delta} \in (-\pi/2, \pi/2)$. The boundedness of the steering angle is true for most wheeled vehicles, particularly in the case of forklifts.

Let $\Omega = \{(X,Y,\theta) : \rho,\phi,\alpha \in R\}$ be the set of all accessible configurations of the vehicle in the configuration space. Let $\Omega_l = \{(X,Y,\theta) : \rho(X,Y) < \varepsilon_{\mathrm{P}} \cap |\phi(X,Y) - \alpha(X,Y,\theta)| < \varepsilon_{\theta_e}\}$ be defined as the local configuration set of the vehicle close to the goal configuration where ε_{P} and ε_{θ_e} are designed as small values. Let $\Omega_g = \Omega - \Omega_l$ be the global configuration set of the vehicle distant from the goal configuration.

The goal of this paper is to establish a stabilization control law for the wheeled vehicle that is robust against input disturbances and measurement noises. The control law is derived based on a hybrid feedback control acting on the configuration sets Ω_l and Ω_g . In the next section, we will derive the control law that renders the initial condition in the global configuration set Ω_g to the local configuration set Ω_l . Then, we will show that the control law derived from the global configuration set Ω_g does not asymptotically stabilize the system when the initial condition is in the local configuration set Ω_l . Afterwards, we will design the control law such that the origin of the system is asymptotically stable in the local configuration set Ω_l .

3. Controller design

3.1 Stabilization in the global configuration set

In this section, we derive the control law for the global set Ω_g . Let the Lyapunov function for the global configuration set Ω_g be given by

$$V_g = V_{g1} + V_{g2} = \rho^2/2 + (k_{\phi}\phi^2 + \alpha^2)/2 , \quad (10)$$

where the terms V_{g1} and V_{g2} represent the squared norms of the navigation variable ρ and the squared weighted norm of the navigation variables ϕ and α , respectively. Let the driving velocity input with the state feedback disturbances ε_{ρ} and ε_{α} be given as follows.

$$v_{dr} = k_{vdr}(\rho + \varepsilon_{\rho})\cos(\alpha + \varepsilon_{\alpha}) , \quad (11)$$

where k_{vdr} is a positive constant gain. The term $\dot{V}_{g1}$ becomes

$$\begin{aligned} \dot{V}_{g1} = (k_{vdr}(-\rho^2\cos^2\alpha - \varepsilon_{\rho}\rho\cos^2\alpha + \rho^2\varepsilon_{\alpha}/2\ \sin 2\alpha) \\ - \varepsilon_{vdr}\rho\cos\alpha)\cos(\delta + \varepsilon_{\delta}). \end{aligned} \quad (12)$$

From (12), we can choose a sufficiently large gain k_{vdr} to neglect the input disturbance ε_{vdr} . In the global configuration set Ω_g , the term $\rho^2\cos^2\alpha$ is more dominant than the term $\varepsilon_{\rho}\rho\cos^2\alpha$. The first term becomes $\dot{V}_{g1} \le 0$ in the region of Ω_g which implies that the term V_{g1} converges to a nonnegative finite limit. Consequently, ρ goes to a small value.

Let the steering angle input with the state feedback disturbances ε_{ρ} , ε_{ϕ} , and ε_{α} and the input disturbance ε_{vdr} be given as follows.

$$\begin{aligned} \delta_g = -\mathrm{atan}(l(k_{\alpha}(\alpha + \varepsilon_{\alpha})/(v_{dr} + \varepsilon_{vdr}) + ((\alpha + \varepsilon_{\alpha}) \\ + k_{\phi}(\phi + \varepsilon_{\phi}))\sin(\alpha + \varepsilon_{\alpha})/((\rho + \varepsilon_{\rho})(\alpha + \varepsilon_{\alpha})))). \end{aligned} \quad (13)$$

By using (13), the term $\dot{V}_{g2}$ becomes

$$\begin{aligned} \dot{V}_{g2} = (-k_{\alpha}\alpha^2 - k_{\alpha}\alpha\varepsilon_{\alpha} - ((k_{\phi}\phi + \alpha)(\varepsilon_{\alpha}\cos\alpha \\ - (\varepsilon_{\rho}/\rho + \varepsilon_{\alpha}/\alpha)\sin\alpha) + (k_{\phi}\varepsilon_{\phi} + \varepsilon_{\alpha})\sin\alpha \\ - \varepsilon_{\delta}\rho\alpha/l)(v_{dr} + \varepsilon_{vdr})/\rho)\cos(\delta + \varepsilon_{\delta}). \end{aligned} \quad (14)$$

From (14), we can choose a sufficiently large gain k_{α} to neglect the disturbance in the third term. In the global set Ω_g , the first term $k_{\alpha}\alpha^2$ is more dominant than the second term $k_{\alpha}\alpha\varepsilon_{\alpha}$. The second term becomes $\dot{V}_{g2} \le 0$ which implies V_{g2} converges to a nonnegative finite limit. Thus, α goes to a small value.

We have shown that when the vehicle is in the configuration set Ω_g , the derivative of the Lyapunov function $\dot{V}_g \le 0$ becomes semi-definite negative. Now, we investigate the implementation of the control law (11) and (13) to the system (7)-(9):

$$\begin{aligned} \dot{\rho} = (k_{vdr}(-\rho\cos^2\alpha - \varepsilon_{\rho}\cos^2\alpha + \varepsilon_{\alpha}\rho\sin 2\alpha/2) \\ - \varepsilon_{vdr}\cos\alpha)\cos(\delta + \varepsilon_{\delta}), \end{aligned} \quad (15)$$

$$\begin{aligned} \dot{\phi} = (k_{vdr}(\sin 2\alpha/2 + \varepsilon_{\rho}\sin 2\alpha/(2\rho) - \varepsilon_{\alpha}\sin^2\alpha) \\ + \varepsilon_{vdr}/\rho\sin\alpha)\cos(\delta + \varepsilon_{\delta}), \end{aligned} \quad (16)$$

$$\begin{aligned} \dot{\alpha} = (-k_{\alpha}\alpha - k_{vdr}k_{\phi}\phi\sin 2\alpha/(2\alpha) - \varepsilon_{\rho}k_{vdr}k_{\phi}\phi\sin 2\alpha/(2\rho\alpha) \\ - \varepsilon_{\alpha}(k_{\alpha} + k_{vdr}k_{\phi}\phi/\alpha + k_{vdr}\cos^2\alpha + \sin\alpha/(\rho\alpha)) \\ - \varepsilon_{\phi}k_{\phi}\sin\alpha/(\rho\alpha) - \varepsilon_{vdr}k_{\phi}\phi\sin\alpha/(\rho\alpha) \\ + \varepsilon_{\delta}k_{vdr}\rho/l\cos\alpha)\cos(\delta + \varepsilon_{\delta}). \end{aligned} \quad (17)$$

In (17), the value of ϕ converges to a small value as the values of ρ and α converge to small values. As a result, by using the control law (11) and (13), the vehicle, which initially starts from the global configuration set Ω_g , will be rendered to the local configuration set Ω_l . The controller derived from the Lyapunov function V_g of the global configuration set Ω_g is rewritten as follows:

$$\begin{aligned} v_{dr} = k_{vdr}\rho\cos\alpha, \\ \delta = -\mathrm{atan}(l(k_{\alpha}\alpha/(k_{vdr}v_{dr}) + (\alpha + k_{\phi}\phi)\sin\alpha/(\rho\alpha))). \end{aligned} \quad (18)$$

3.2 The unstabilized system in the local configuration set

In this section, we show that the control law derived from the global configuration set Ω_g does not asymptotically stabilize the system in the local configuration set Ω_l. In the previous section, by using (18), we have shown that the initial configuration of the vehicle that started from the global configuration Ω_g set will go to the local configuration set Ω_l. Therefore, we assume that the navigation variable ρ goes to a small parameter ε_{P} (note that $\varepsilon_{\mathrm{P}} > 0$ as ρ is always positive). Also, we assume that the navigation variables ϕ and α go to small values close to the state feedback disturbances ε_ϕ and ε_α, respectively. The approximation of the system (15)-(17) near the origin where the initial values of ρ, ϕ, and α are almost equal to ε_{P}, ε_ϕ, and ε_α, respectively, is given by

$$\dot{\rho} = (k_{vdr}(-\varepsilon_{\mathrm{P}} - \varepsilon_\rho) - \varepsilon_{vdr})\cos(\delta + \varepsilon_\delta), \tag{19}$$

$$\dot{\phi} = k_{vdr}\varepsilon_\alpha(1 + \varepsilon_\rho / \varepsilon_{\mathrm{P}})/2\cos(\delta + \varepsilon_\delta), \tag{20}$$

$$\begin{aligned}\dot{\alpha} = (&-(k_{vdr} + 2k_\alpha)\varepsilon_\alpha - 2k_{vdr}k_\phi\varepsilon_\phi \\ &-(k_\phi\varepsilon_\phi + \varepsilon_\alpha)/\varepsilon_{\mathrm{P}})\cos(\delta + \varepsilon_\delta).\end{aligned} \tag{21}$$

By choosing the small parameter $\varepsilon_{\mathrm{P}} \geq |\varepsilon_\rho| + |\varepsilon_{vdr}| / k_{vdr}$, the derivative of the Lyapunov function V_{g1} at the boundary between the global and the local configuration sets is derived as follows.

$$\dot{V}_{g1} = (k_{vdr}(-\varepsilon_{\mathrm{P}}^2 - \varepsilon_\rho\varepsilon_{\mathrm{P}}) - \varepsilon_{vdr}\varepsilon_{\mathrm{P}})\cos(\delta + \varepsilon_\delta) \leq 0. \tag{22}$$

In (22), V_{g1} is bounded; thus, ρ is also bounded. Therefore, the control law (11) can be used to stabilize the subsystem (4).

It is clear that subsystem (20) has a finite escape time when $\varepsilon_\alpha(1 + \varepsilon_\rho / \varepsilon_{\mathrm{P}}) > 0$. Subsystem (21) will exhibit the same condition as subsystem (20) as α is proportionally related to the variable ϕ. In the rest of the paper, we will use the driving velocity control law (11) and re-design the steering angle control law (13) to obtain the robustness property of the closed loop system (1)-(3).

3.3 Stabilization in the local configuration set

For the local configuration set Ω_l, let a Lyapunov function $V_l : R^3 \to R$ be given as

$$V_l = (\rho^2 + (\phi - \alpha)^2)/2. \tag{23}$$

Since $\theta_e = \phi - \alpha$, we have

$$\begin{aligned}\dot{\theta}_e &= \dot{\phi} - \dot{\alpha}, \\ &= -k_{vdr}(v_{dr} + \varepsilon_{vdr})\tan(\delta + \varepsilon_\delta)/l\cos(\delta + \varepsilon_\delta).\end{aligned} \tag{24}$$

In (24), the disturbances caused by the driving velocity input and state feedback disturbances are eliminated. Let the control law of the steering angle for the local configuration set Ω_l be given as follows.

$$\delta_l = \operatorname{atan}(lk_{\theta_e}\theta_e / (k_{vdr}v_{dr})), \tag{25}$$

where k_{θ_e} is a positive gain. It has been verified in Section 3.1 that control law (18) renders the variables ρ, ϕ, and α to the boundary of set Ω_l. By choosing the small parameter $\varepsilon_{\theta_e} \geq ((k_\phi + 1)|\varepsilon_\alpha| + \varepsilon_{\mathrm{P}}|\varepsilon_\delta|)/k_\phi$, the derivative of Lyapunov function V_{g2} at the boundary between the configuration sets Ω_g and Ω_l is as follows.

$$\begin{aligned}\dot{V}_{g2} = (&-k_\alpha\varepsilon_\alpha^2 - k_\alpha\varepsilon_\alpha^2\operatorname{sgn}(\varepsilon_\alpha) - k_{vdr}(k_\phi\varepsilon_{\theta_e}\varepsilon_\alpha \\ &+ (k_\phi + 1)\varepsilon_\alpha^2 - \varepsilon_{\mathrm{P}}\varepsilon_\alpha\varepsilon_\delta))\cos(\delta + \varepsilon_\delta) \leq 0.\end{aligned} \tag{26}$$

In the local configuration set Ω_l, the initial configuration of θ_e starts from ε_{θ_e}. Then, the derivative of Lyapunov function (23) is as follows.

$$\dot{V}_l = ((k_{vdr}(-\varepsilon_{\mathrm{P}}^2 - \varepsilon_\rho\varepsilon_{\mathrm{P}}) - \varepsilon_{vdr}\varepsilon_{\mathrm{P}} - k_\theta\varepsilon_{\theta_e}^2)\cos(\delta + \varepsilon_\delta) \leq 0. \tag{27}$$

This implies that the configuration of the vehicle will not escape to the larger values of ε_{P} and ε_{θ_e} when the vehicle configuration is in the set Ω_l. The novelty of the robust stabilization control against the input and measurement disturbances for the wheeled vehicle system is rewritten as follows.

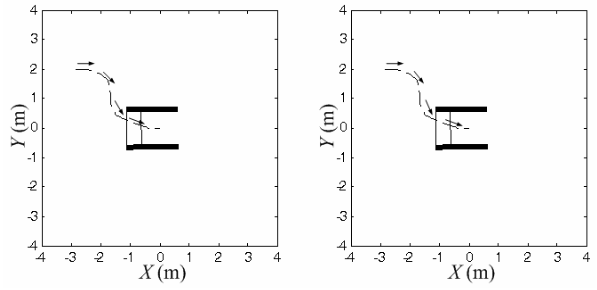
$$\begin{aligned} v_{dr} &= k_{vdr}\rho\cos\alpha, \\ \delta &= \begin{cases} -\operatorname{atan}(l(k_\alpha\alpha / k_{vdr}v_{dr} + (\alpha + k_\phi\phi)\sin\alpha/(\rho\alpha))) \\ \qquad\qquad\qquad\qquad \text{when} \quad \Omega_g, \\ \operatorname{atan}(k_\theta l\theta_e / (k_{vdr}v_{dr})) \qquad \text{when} \quad \Omega_l. \end{cases}\end{aligned} \tag{28}$$

4. Simulation and experiment

4.1 Simulation results

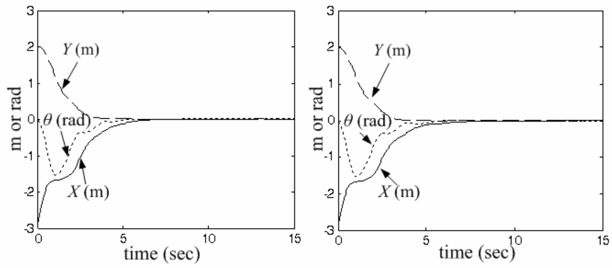
In this section, the behaviors of the control law (18) derived from the Lyapunov function in the global configuration set Ω_g and the hybrid feedback controller (28) derived from the two Lyapunov functions in both configuration sets Ω_g and Ω_l are investigated. The configuration of the vehicle is presented by (X m, Y m, θ rad). In the simulations, the initial and the goal configurations of the vehicle are set at (-3, 2, 0) and (0, 0, 0), respectively.

In the first simulation, the measurement noises and input disturbances are regarded as random distribution values. The absolute maximum values of measurement noises and input disturbances are as follows: $\varepsilon_X^{\max} = \varepsilon_Y^{\max} = \varepsilon_\theta^{\max} = 0.01$; $\varepsilon_{vdr}^{\max} = \varepsilon_\delta^{\max} = 0.01$. For both controllers (18) and (28), the parameters are set as follows: $k_{vdr} = 1$, $k_\alpha = 10$, and $k_\phi = 10$. For the controller (28), the parameters are set as follows: $\varepsilon_{\mathrm{P}} = \varepsilon_{\theta_e} = 0.02$ and $k_{\theta_e} = 1$. The stabilization results in the X-Y plane are presented in Fig. 2. Figs. 3 and 4 show the trajectories of position and orientation (X, Y, θ) and the control inputs (v_{dr}, δ), respectively, of the stabilization results with the assumption of random measurement noises and input disturbances. In Figs. 2(a) and (b), there is not much difference between the controller (18) and (28) as both controllers (18) and (28) drive the vehicle to the final goal configuration. The difference emerges in the steering angle control signals (Fig. 4). In Fig. 4(a), by using controller (18), the steering angle



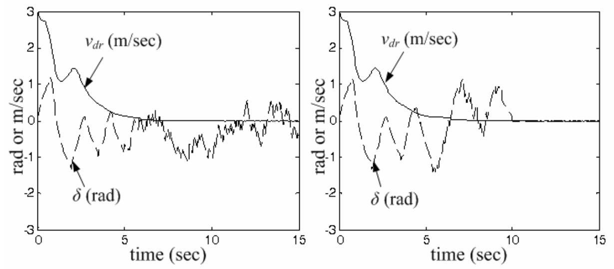
(a) Using control law (18) (b) Using control law (28)

Fig. 2. Stabilization results with the assumption of random measurement noises and input disturbances.



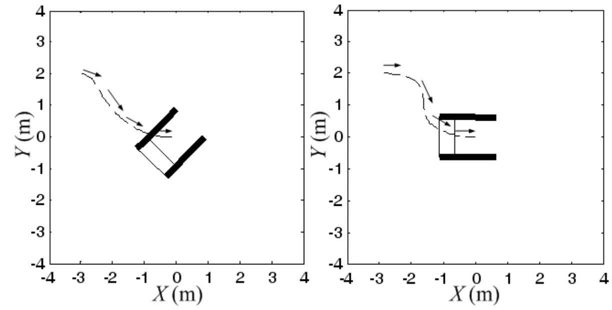
(a) Using control law (18) (b) Using control law (28)

Fig. 3. Trajectories of *X*, *Y*, and θ of the stabilization with the assumption of random measurement noises and input disturbances.



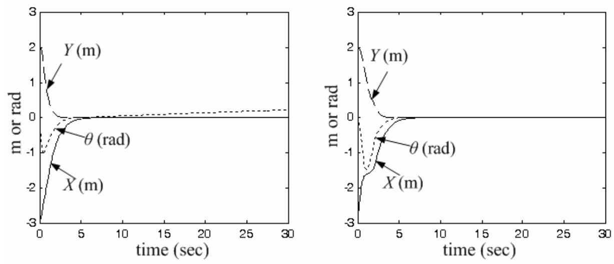
(a) Using control law (18) (b) Using control law (28)

Fig. 4. Trajectories of v_{dr} and δ of the stabilization with the assumption of random measurement noises and input disturbances.



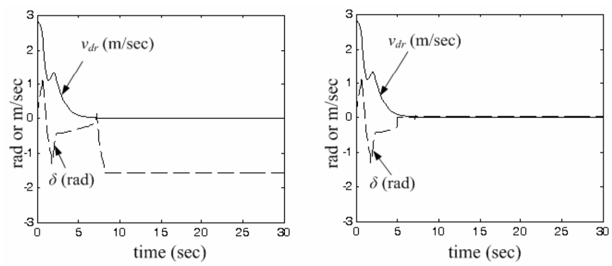
(a) Using control law (18) (b) Using control law (28)

Fig. 5. Stabilization results with the assumption of constant measurement noises and input disturbances.



(a) Using control law (18) (b) Using control law (28)

Fig. 6. Trajectories of *X*, *Y*, and θ of the stabilization with the assumption of constant measurement noises and input disturbances.



(a) Using control law (18) (b) Using control law (28)

Fig. 7. Trajectories of v_{dr} and δ of the stabilization with the assumption of constant measurement noises and input disturbances.

signal δ oscillates due to the random disturbances. In Fig. 4(b), by using the hybrid feedback controller (28), the steering angle signal δ slightly oscillates after the switching of the controller from the global to the local configuration set.

In the second simulation, we will show that the hybrid feedback control law rejects the constant measurement noises and input disturbances that might appear in practice [29]. The measurement noises and input disturbances are given as constant signals as follows: $\varepsilon_X = \varepsilon_Y = \varepsilon_\theta = 0.01$; $\varepsilon_{vdr} = \varepsilon_\delta = 0.01$. The parameters are set as in the first simulation. In Fig. 5(a), although the coordinates (*X*, *Y*) go to zero, the controller (18) cannot handle the orientation θ in the local configuration set Ω_l due to the small input disturbances and measurement noises. The vehicle keeps turning in its position. Conversely, the proposed controller (28) ensures the vehicle configuration is maintained in the local configuration set near the origin. The trajectories of position and orientation (*X*, *Y*, θ) and the control inputs (v_{dr}, δ) of the stabilization results with the assumption of constant measurement noises and input disturbances are given in Figs. 6 and 7, respectively.

4.2 Experiment results

The developed robust stabilization controller (28) was implemented using the autonomous forklift shown in Fig. 8. The distance between the center of the rear wheel and the axis of the front wheels *l* was 1.2 m. The vehicle had two ac motors: driving and steering motors. The two motors were connected to the programmable logic controller (PLC) which serves as a low-level controller. The PLC implements a digital proportional-integral-derivative (PID) controller to the motors with a cycle time 10 ms. The control algorithm for the vehicle stabilization was programmed in C++ and ran with a sampling

Fig. 8. Autonomous forklift used in the experiment.

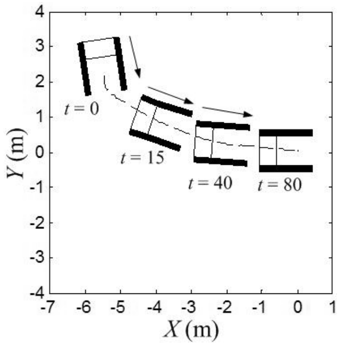
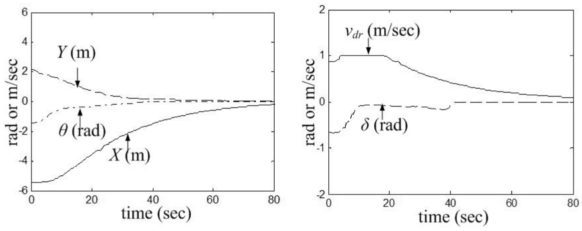


Fig. 9. Experiment result: X-Y coordinates.



(a) X, Y, and θ (b) v_{dr} and δ

Fig. 10. Trajectories of the configuration (X, Y, θ) and the control inputs (v_{dr}, δ) of the experiment.

time of 100 ms running on an industrial PC (Pentium 1.4 GHz) with a Windows XP operating system. The PLC communicated with the industrial PC via RS232 communication. The forklift was equipped with laser-based localization sensor NAV200 which provides the measurement of position and orientation. The NAV200 was connected to the industrial PC via RS232 communication. The localization sensor had 8 mm positioning accuracy and 0.1^o angular accuracy. The range of the steering angle was set to $|\delta| \leq$ 1.48 rad (85^o) and the maximum value of the driving velocity control input for the vehicle was set to $|v_{dr}| \leq$ 1 m/s.

The vehicle was initially located at (X(0), Y(0), θ(0)) = (-5.47, 2.19, -1.414). The parameters were set as follows: $\varepsilon_P = 0.1$, $\varepsilon_{\theta_e} = 0.08$, $k_{vdr} = 0.5$, $k_\alpha = 5$, $k_\phi = 5$, and $k_{\theta_e} = 1$. Figs. 9 and 10 show the trajectories of the path in the X-Y coordinate, the configuration (X, Y, θ), and the input controls v_{dr} and δ with respect to time. From the experiment, the accuracy of the vehicle configuration at the final time $t_f = 80$ sec was ($X(t_f)$, $Y(t_f)$, $\theta(t_f)$) = (0.1, 0.05, 0.01). It was observed that the vehicle was driven to the origin and the robustness of the system was verified wherein the system did not escape from the local configuration set due to the input disturbances and the measurement noises.

5. Conclusions

In this paper, a robust stabilization control problem of a wheeled vehicle was formulated in the presence of input disturbances and measurement noises. It was shown that small input disturbances and measurement noises could destroy the global stability of a wheeled vehicle system, especially in the local area near the origin. A robust stabilization of the wheeled vehicle was designed through a hybrid feedback control scheme in which additive input disturbances and measurement noises were included in the control design. The presented robust control yielded a switching feedback control law acting on the global and local configuration sets of the vehicle configuration. Asymptotical convergence and robustness properties of the system were investigated using Lyapunov analysis. The resulting control law was validated by simulations of and experiments in the stabilization of an autonomous forklift.

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